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Independent Junior Research Group for Bayesian Statistics

### Multilevel Models

Grouped data

$$\mu_n = \sum_{i=1}^p x_{ni} b_i + \sum_{i=1}^p x_{ni} \left( \sum_{g \in G_i} u_{ig|n_i} \right)$$

•  $b_i$  Overall coefficients  
•  $u_{ig}$  Varying coefficients

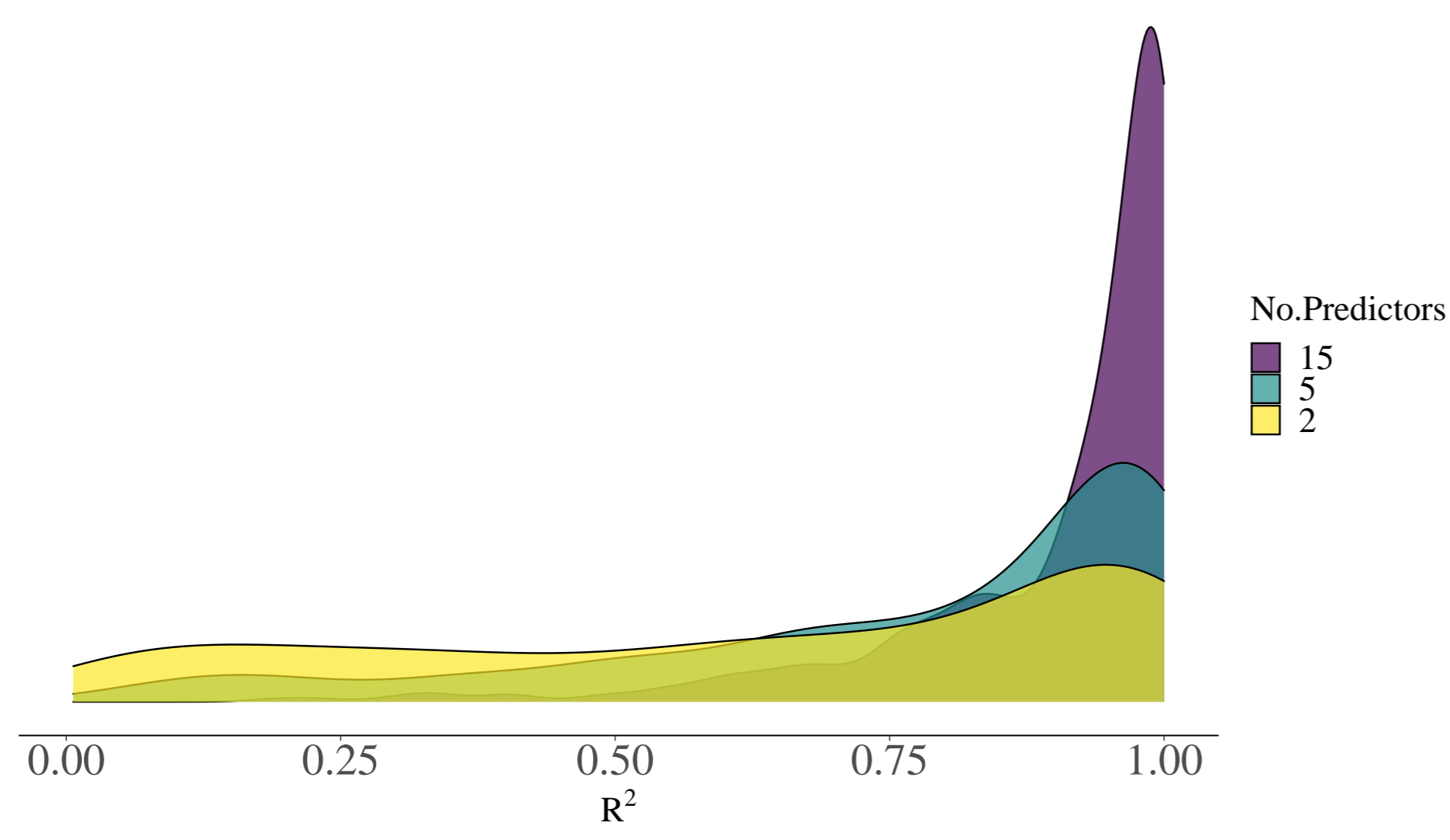
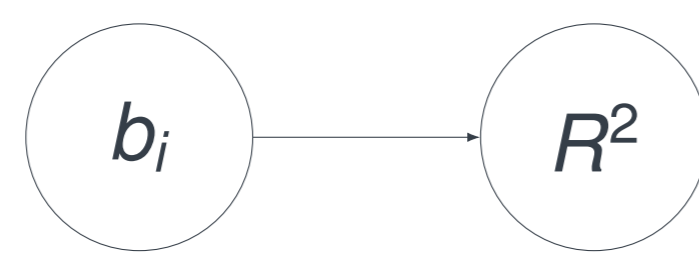
$$y_n \sim N(\mu_n, \sigma)$$

### Common Inference Setting

Weakly informative priors

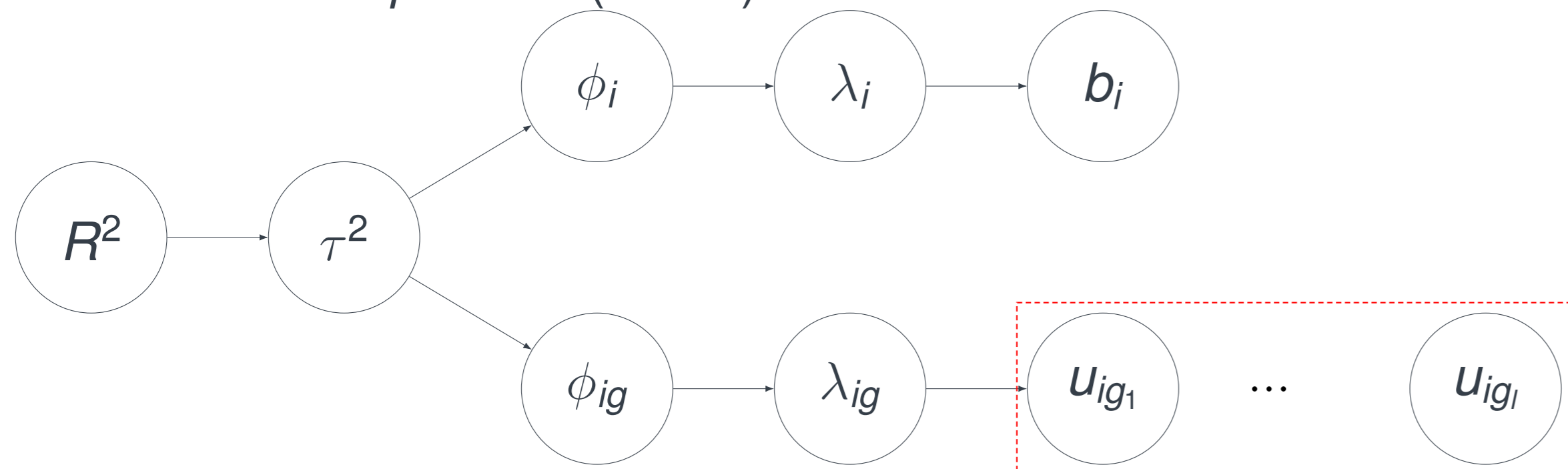
$$b_i \sim N(0, 1), \sigma \sim E(1)$$

Effect on proportion of explained variance



### An intuitive joint prior on $R^2$

Specify a prior on  $R^2$  and decompose the explained variance via a Dirichlet Decomposition (R2D2)

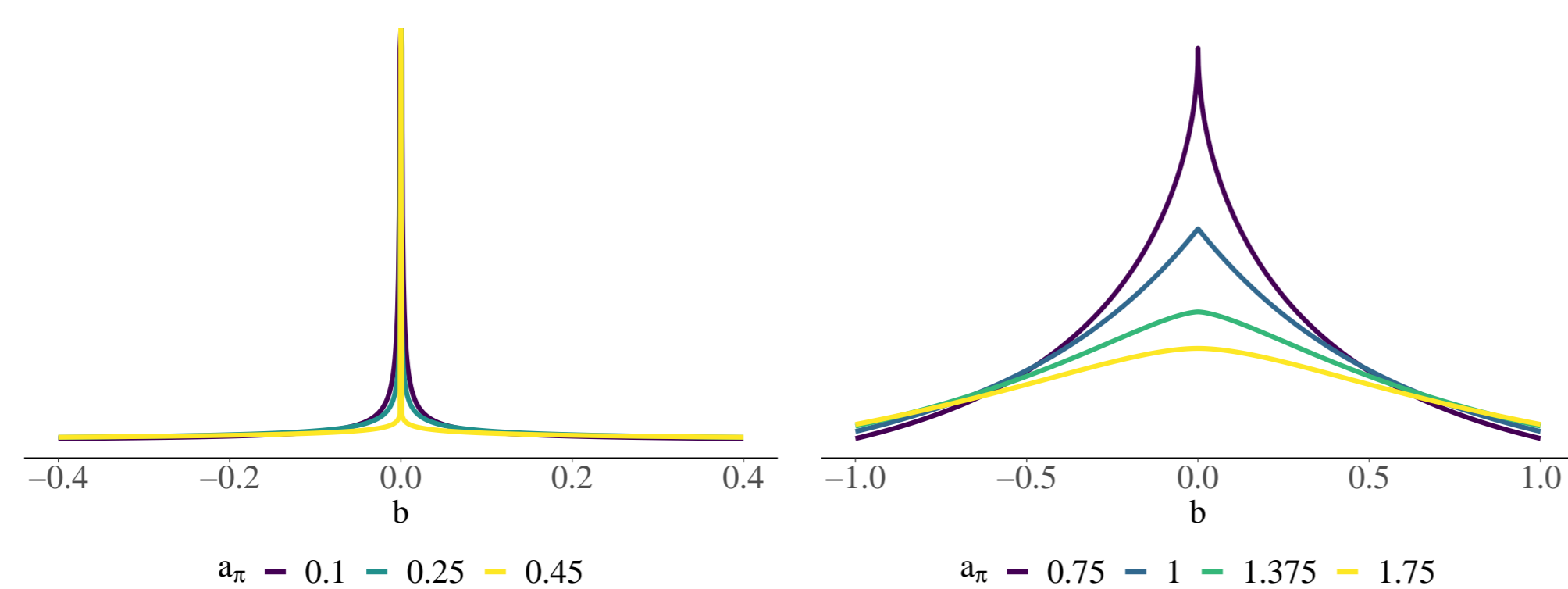


$R^2$  Dirichlet Decomposition Multilevel Models: R2-D2-M2 prior

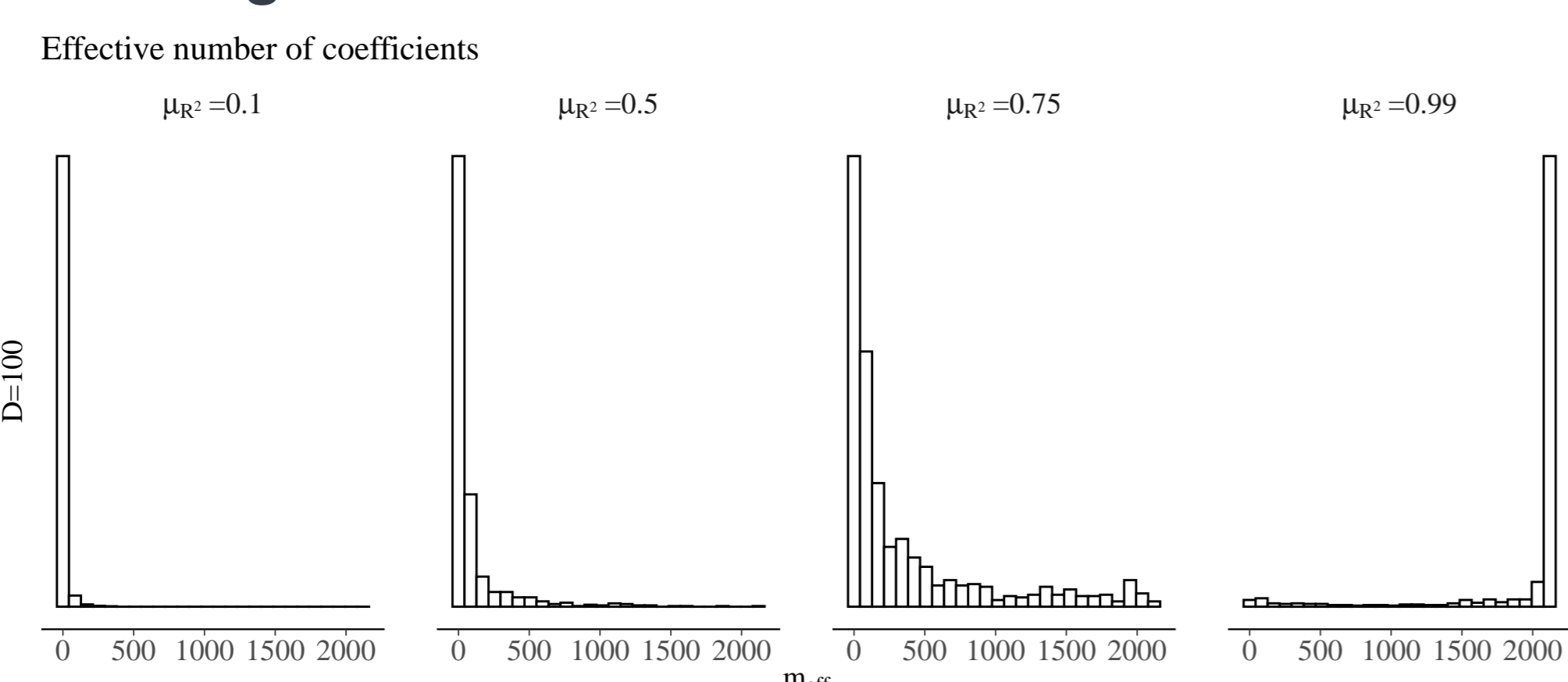
$$R^2 \sim B(\mu, \varphi), \phi \sim \text{Dirichlet}(\alpha), b_i \sim N(0, \lambda_i), u_{ig} \sim N(0, \lambda_{ig}), \sigma \sim p(\sigma)$$

### Properties

#### Marginal Priors

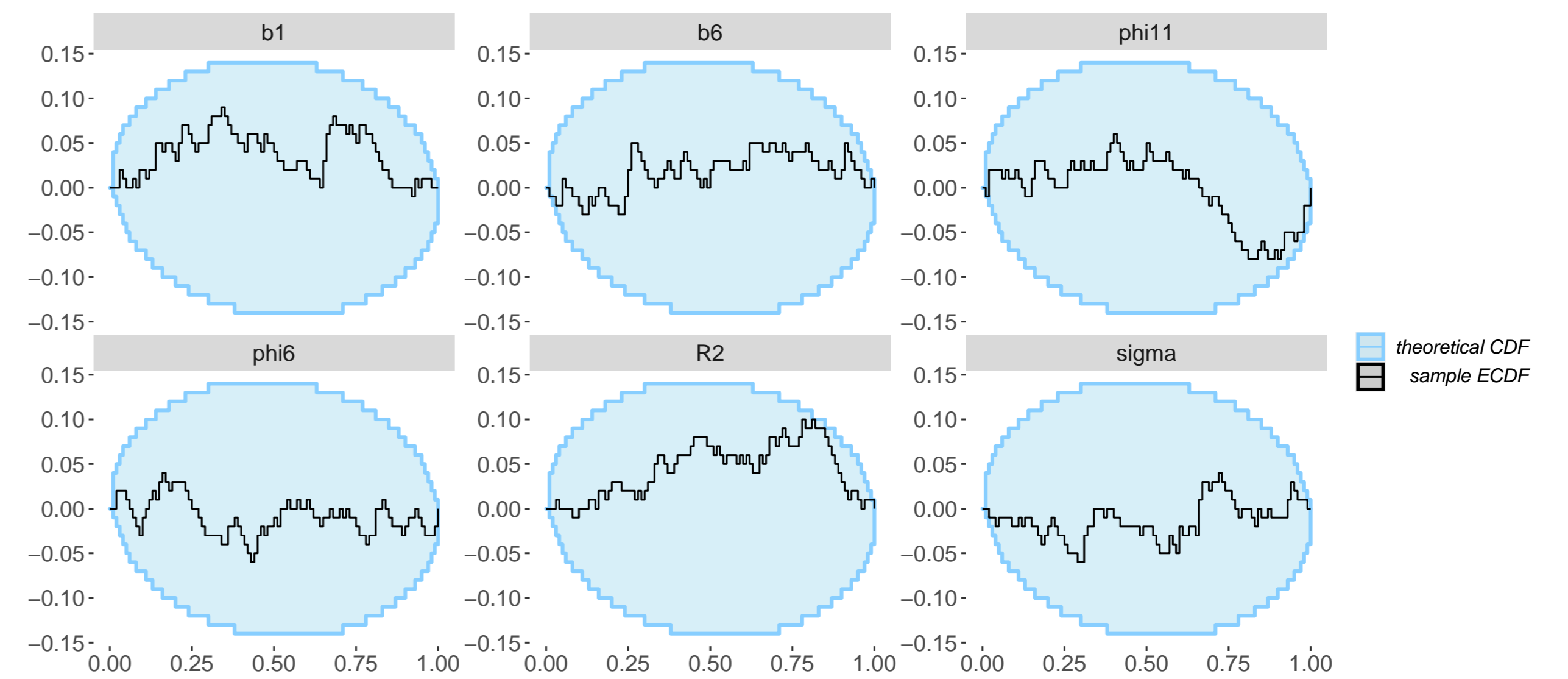


#### Implied shrinkage



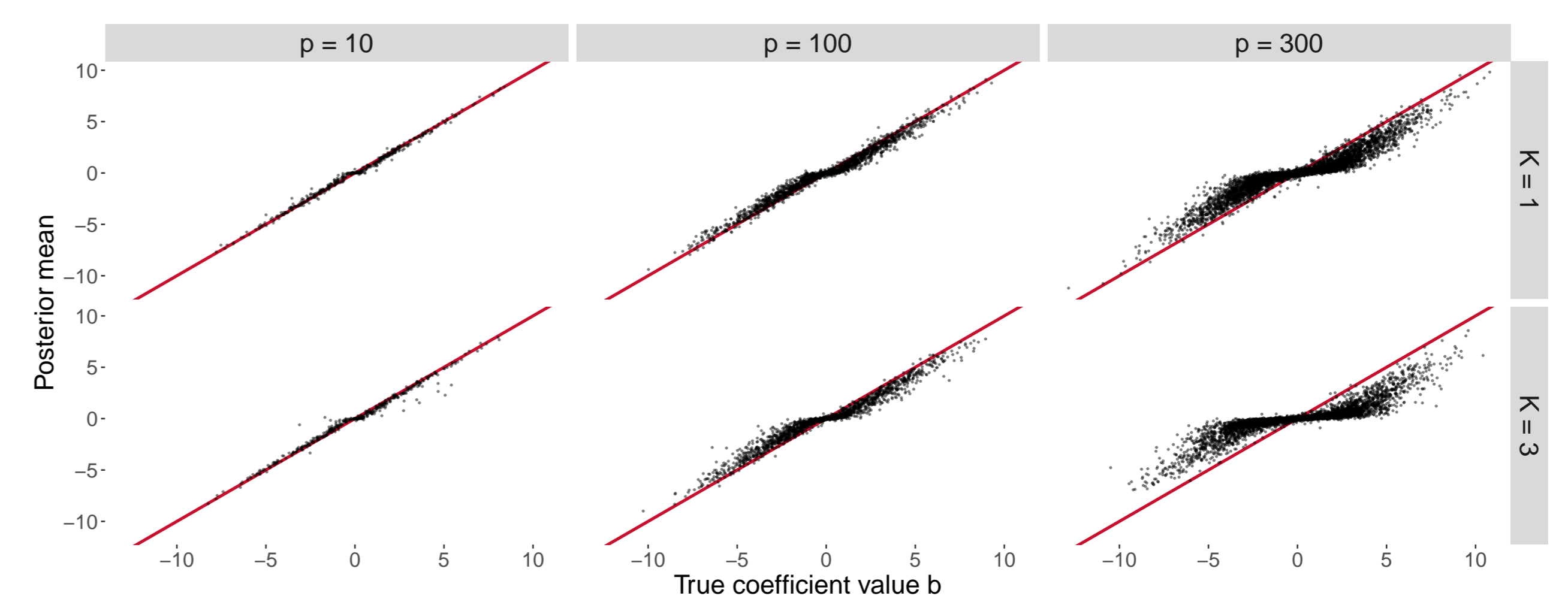
### Experiments

#### Simulation Based Calibration

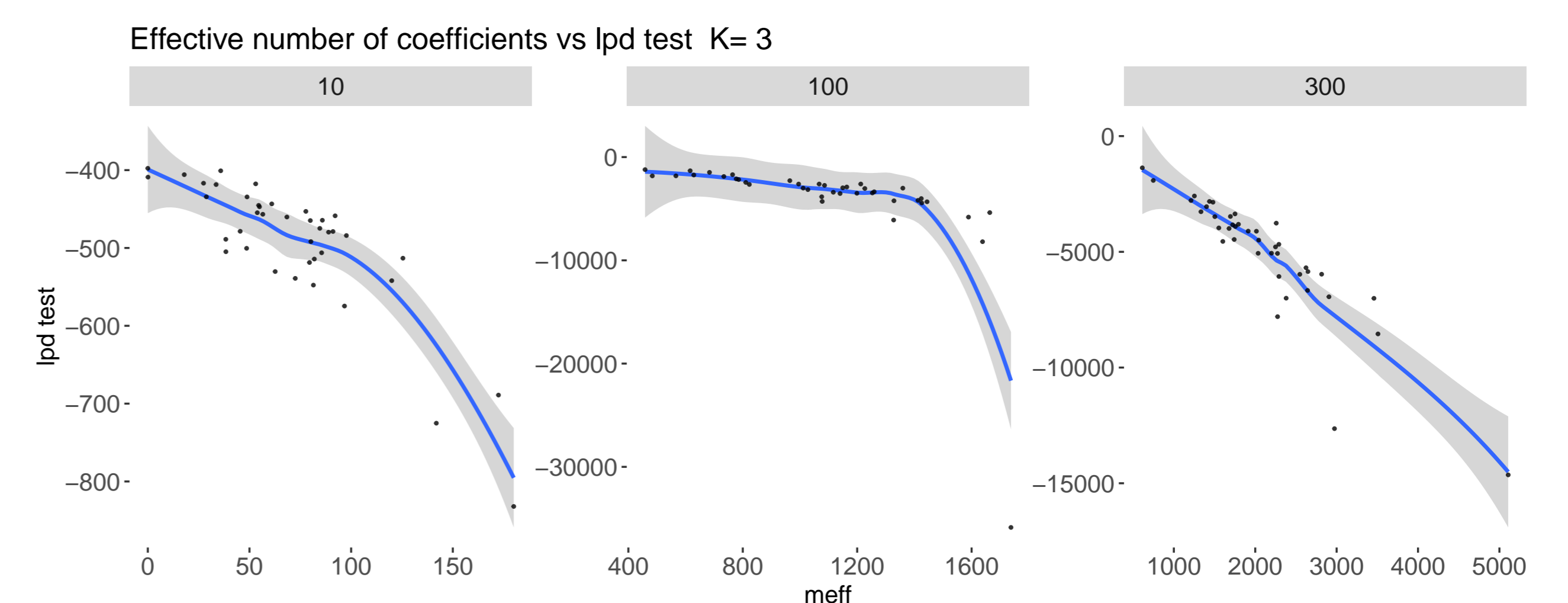


#### General Multilevel Model Simulation

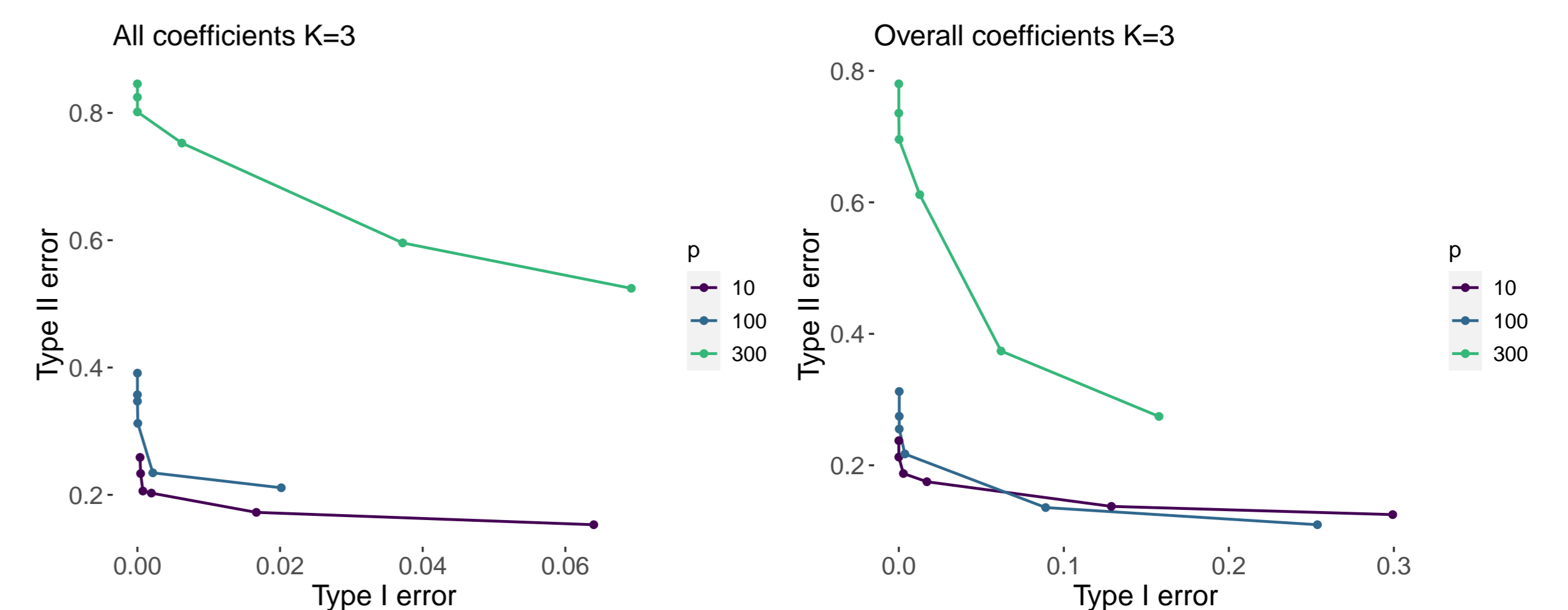
##### Posterior shrinkage



##### Out-of-sample Predictive Performance



##### Posterior coverage and Credibility Intervals



### Conclusions

- Our model is well calibrated.
- Prior specification over the whole set of coefficients presents advantages.
- Out-of-sample predictive performance is related to shrinkage.
- Errors are properly controlled.